Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Winter Examination-2020

Subject Name: Topology

Subject Code: 5SC01TOP1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 12/03/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1		**Attempt the Following questions**	**(07)**
	a.	Define: Topology	**1**
	b.	Let $X = R$ and $A = (0,1]$. Find A' and $Bd(A)$.	**2**
	c.	Let A, B denotes subsets of a topological space X. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$.	**2**
	d.	Give an example which shows $(A \cup B)^0 \neq A^0 \cup B^0$.	**2**
Q-2		**Attempt all questions**	**(14)**
	a.	Let X be a set. Let $\tau_f = \{U \subset X \mid \text{either} X - U = X \text{or} X - U \text{is finite}\}$ then prove that (X, τ_f) is topological space.	**5**
	b.	Let (X, τ) be a topological space and Y be a non empty subset of X then prove that the collection $\tau_y = \{U \cap Y \mid U \in \tau\}$ is a topology on Y.	**5**
	c.	Let Y be a subspace of X then prove that a set A is closed in Y if and only if $A = Y \cap C$ where C is closed in X.	**4**
		OR	
Q-2		**Attempt all questions**	**(14)**
	a.	Let X, Y be topological spaces. $f: X \to Y$. Then prove that following are equivalent (1) f is continuous. (2) for every subset A of X, then $f(\bar{A}) \subset \overline{f(A)}$ (3) For every closed set B of Y, $f^{-1}(B)$ is closed in X.	**6**
	b.	Let (X, d) be a metric space. Define $\tau = \{U \mid for every x \in U there exist \epsilon > 0 \ such that B_\epsilon(x) \subset U\}$. Then prove that τ is topology on X.	**5**
	c.	Let R and R_l be set of real numbers with usual topology and lower limit topology respectively. Let $f: R_l \to R$, by $f(x) = x$. Is f continuous function? Justify your answer.	**3**

Q-3		**Attempt all questions**	**(14)**
	a.	Let (X_1, τ_1) and (X_2, τ_2) be topological spaces. Let $X = X_1 \times X_2$. Define $\beta = \{U_1 \times U_2 \mid U_1 \in \tau_1, U_2 \in \tau_2\}$. Prove that β is a basis for X.	**5**
	b.	Let A be a subset of topological space X and A' be the set of all limit points of A. Then prove that $\bar{A} = A \cup A'$.	**5**
	c.	State and prove Pasting Lemma.	**4**

OR

Q-3		**Attempt all questions**	**(14)**
	a.	Prove that continuous image of compact space is compact.	**5**
	b.	State and prove Sequence lemma.	**5**
	c.	Let X be a topological space and $A \subset X$. Then prove that $x \notin \bar{A}$ if and only if there exists an open set U containing x that does not intersect A.	**4**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Product of two normal space is normal. True/False.	1
	b.	Define: Compact Space	2
	c.	Define: Second Countability Axiom	2
	d.	State Urysohn's lemma	2

Q-5		**Attempt all questions**	**(14)**
	a.	Let X and Y be two topological spaces. Let $f: X \to Y$ be continuous function. If every convergent sequence $x_n \to x$ in X then prove that $f(x_n) \to f(x)$ in Y. Also prove that converse is hold if X is metrizable.	**6**
	b.	Prove that a topological space (X, τ) is T_1 space if and only if $\{x\}$ is closed $\forall\, x \in X$.	**5**
	c.	Give an example of topological space which is T_1 space but not T_2 space.	**3**

OR

Q-5		**Attempt all questions**	**(14)**
	a.	Let X be T_1 space and $A \subset X$. Then prove that $x \in A'$ if and only if for each open set U of $x, U \cap A$ is infinite.	**5**
	b.	Prove that every closed subset of compact set is compact.	**5**
	c.	Let X be a co countable topological space then prove that $X$is compact if and only if X is finite	**4**

Q-6		**Attempt all questions**	**(14)**
	a.	Prove that every subspace of T_2space is T_2 space.	**5**
	b.	Let X be topological space then prove that X is disconnected if and only if there exists a non empty proper subset of X which is both open and closed.	**5**
	c.	Prove that continuous image of connected space is connected.	**4**

OR

Q-6		**Attempt all Questions**	**(14)**
	a.	State and prove Heine - Borel theorem	**10**
	b.	Consider R with lower limit topology then prove that R is disconnected.	**04**